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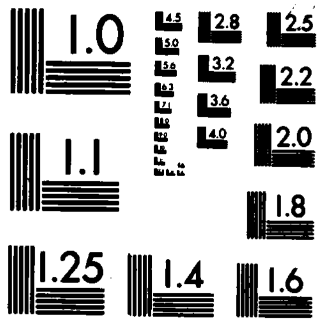
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THE THEORY OF AN ATTITUDE AND POSITION MEASURING TECHNIQUE USING TWO CAMERAS INCORPORATING LINEAR CHARGE COUPLED IMAGING ARRAYS

by

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SUMMARY

The theory is presented of a technique for measuring the attitude and position of cylindrical objects which is of interest in studying the behaviour of stores ejected from aircraft wing pylons. The sight-line angles of the tangents to the top and bottom edges of the store are measured in two vertical planes using linear CCD array cameras, and equations relating these angles to the required attitude and position are derived in this Report. The measurement of bank angle from the observation of marks painted on the side of the store, is also considered. The errors in position and attitude determination, which arise from uncertainties in the angle measurements and camera geometry, are discussed and simple first order approximations are derived for these.

Departmental Reference: IT 178

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1 INTRODUCTION

A new method of measuring the position and attitude of cylindrical stores during their release or ejection from aircraft wing pylons is currently being investigated by Instrumentation and Trials Department RAE. The aim is to make available, within a few minutes of store release, information such as the pitch rate and separation velocity time-histories, thereby eliminating the lengthy time scales and tedious analysis associated with existing photographic techniques. Initial interest is focussed on the ground testing of ejector performance but the wider problems of airborne carriage and release trials are being kept firmly in mind.

The technique being studied employs two solid state cameras incorporating linear charge coupled imaging devices (CCDs) to measure the angles of the sight-line tangents to the top and bottom edges of the store, in two separate vertical planes. These four measurements, together with a knowledge of the store radius and camera positions, enable the position of the store in two dimensions and its heading and elevation attitude to be determined, provided that the cross-section of the store is cylindrical in the measurement zone. It follows from this restriction that measurement of the sight-line tangent angles is incapable of determining roll about the longitudinal axis of the store, or motion along this direction. In principle however, bank angle may be determined by the observation of suitable markings painted along the side of the store with the same CCD cameras as used for measuring heading and elevation angles.

The aim of this Report is to present the geometrical relationships between the store position and attitude and the measure sight-line tangent angles from which the former may be calculated. Although not required in the present application the computation of bank is also included for completeness. Finally the influences of measurement and alignment errors on the calculated store position and attitude are discussed. The detailed design and results of the evaluation of an experimental system will be reported elsewhere in due course.

An advantage of the technique considered here, in addition to its fast response time, is that no modification of the store is necessary, other than a coat of paint to provide a high contrast image against its background. A major disadvantage however is the need for a cylindrical cross-section at least in the regions viewed by the cameras. In principle this could be overcome by the use of additional cameras and by making use of the known profile of the store, however the computational load would increase considerably and a more versatile approach in this instance would be to use shape correlation algorithms on a complete image of the store. This would probably require a large main-frame computer for the necessary processing speed and power. In contrast, the computation involved in the technique described here is within the capabilities of a hand-held programmable calculator and since at least two of the stores in question do have cylindrical cross-sections, over some part of their profile, the technique clearly merits further consideration.

The term attitude is used in a restricted sense in this Report to include only elevation and heading angles. The third parameter, bank angle, normally encompassed in

the definition of attitude is only referred to in the specific section dealing with the derivation of the bank angle equations (section 2.5).

2 THE GEOMETRICAL EQUATIONS

2.1 General

The basic arrangement of two linear array cameras positioned to one side of the store whose attitude and position are required, is illustrated in Fig 1. Both cameras are accurately aligned so that each CCD array has a fan shaped field of view in a vertical plane which is of sufficient extent to include the top and bottom edges of the store throughout the required displacement range. If the store is cylindrical, then its cross-section in the plane of either camera will be an ellipse, in general, whose orientation and eccentricity are functions of the store's attitude. The angles of the sight-line tangents from the cameras to the elliptical sections will therefore be functions of this attitude as well as relative position.

The two cameras are assumed to lie in the same horizontal plane and the line joining them is conveniently taken to define the datum heading angle relative to which store heading ψ * is to be measured. The cameras need not, of course, be set up to view exactly at right angles to this direction, nor need their viewing planes be parallel to each other, although the geometrical relationships will obviously simplify in these special cases. The general case is considered here however, by specifying separate heading angles for the viewing planes of the two cameras.

In the analysis which follows, the equation of the elliptical section of the store in the plane of one of the cameras is derived first and then the equations of the sight-line tangents to this ellipse from the camera position are determined, giving the sight-line angles as functions of store attitude and position. Manipulation of the sum and differences of the angles to the top and bottom edges of the store enables a set of equations to be derived which express store position as a function of these compound angles and attitude. A further set of equations is derived which link store attitude to store position and the known geometry of the cameras. These two sets of equations must then be solved simultaneously if store attitude and position are to be found independently. Unfortunately it has not proved possible to solve these equations analytically but an interative solution is presented which is suitable for implementation on a computer or programmable calculator and the performance of this technique on a TI59 calculator is discussed in section 3.

The equations required for the calculation of bank angle are derived in section 2.5 and require an *a priori* knowledge of pitch, yaw and position.

2.2 Equation of the store section

The equation of the store section in the camera viewing plane can be found from the intersection of this plane with the three-dimensional equation of the cylindrical surface of the store.

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* For a complete list of symbols see page 34.

If the store has a radius a and an instantaneous attitude described by elevation angle θ and heading ψ, then, for the spherical polar coordinate system shown in Fig 2, the equation of its three-dimensional surface is shown in Appendix A to be:

$$R^2 = \frac{a^2}{1 - \left\{\sin\alpha \sin\theta + \cos\alpha \cos\theta \cos(\beta - \psi)\right\}^2} \tag{1}$$

where R = radius from the origin of a point P on the store surface

α = elevation angle of point P above the horizontal plane

β = bearing of point P in the horizontal plane.

Note that the origin of the coordinate system used here has been chosen to lie on the centre line of the store. If we further assume that the viewing plane also passes through the origin, with heading angle ψ_0, then the store section in this plane is given simply by substituting $\beta = \psi_0$ in equation (1) above. We may also rearrange the trigonometric functions in the denominator of the equation to give finally for the equation of the store section:

$$R^2 = \frac{a^2}{1 - K\sin^2(\alpha + \gamma)} \tag{2}$$

where $K = 1 - \cos^2\theta \sin^2(\psi_0 - \psi)$,

$\gamma = \tan^{-1}\left\{\cot\theta \cos(\psi_0 - \psi)\right\}$.

Appendix B shows that this represents an ellipse with minor and major semi-axes of a and $a/(1 - K)^{\frac{1}{2}}$ respectively, rotated through an angle γ, where γ is the angle between the minor axis of the ellipse and the horizontal plane.

2.3 The sight-line tangents

The angles of the sight-line tangents to the top and bottom edges of the store may now be determined by finding the equations of the tangents to the elliptical store section described by equation (2) above. This operation is facilitated by changing the origin of the coordinate system employed from the centre of the store to the camera position, whereupon the tangents become simple radial vectors passing through the new origin.

If the position of the store centre is specified by range R_0 and depression angle α_0 in the new coordinate system as shown in Fig 3 then the new coordinates of a point P on the surface of the store R', α' may be related to the old coordinates R, α by the equations:

$$\left.\begin{aligned} R\cos\alpha &= R'\cos\alpha' - R_0\cos\alpha_0 \\ R\sin\alpha &= R_0\sin\alpha_0 - R'\sin\alpha' \end{aligned}\right\} \tag{3}$$

The angles α' and α_0 have been defined as depression angles here in anticipation of the likely experimental configuration.

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Transforming the store section into the new coordinate system using equation (3) gives, after some manipulation:

$$R'^2\left\{1 - K\sin^2(\alpha' - \gamma)\right\} - 2R'R_0\left\{\cos(\alpha_0 - \alpha') - K\sin(\alpha_0 - \gamma)\sin(\alpha' - \gamma)\right\}^2$$
$$+ R_0^2\left\{1 - K\sin^2(\alpha_0 - \gamma)\right\} - a^2 = 0 \quad . \qquad (4)$$

This is a quadratic in R' indicating that for a given radius vector at depression angle α' there are in general two points of intersection with the elliptical store section. As the vector approaches a tangent position however the two roots of the equation must converge to a single value and this criterion of equal roots can be used to establish the angles of the tangents.

The roots of a quadratic of the general form $Ax^2 + Bx + C = 0$ are equal when $B^2 = 4AC$, hence from equation (4) above the tangent angles must satisfy:

$$R_0^2\left\{\cos\left(\alpha_0 - \alpha'\right) - K\sin(\alpha_0 - \gamma)\sin(\alpha' - \gamma)\right\}^2$$
$$= \left\{1 - K\sin^2(\alpha' - \gamma)\right\}\left\{R_0^2\left(1 - K\sin^2(\alpha_0 - \gamma)\right) - a^2\right\} \quad . \qquad (5)$$

In order to solve this for the angle α' we can multiply out and make the substitutions:

$$\cos(\alpha_0 - \alpha') = \cos\left\{(\alpha_0 - \gamma) - (\alpha' - \gamma)\right\}$$
$$= \cos(\alpha_0 - \gamma)\cos(\alpha' - \gamma) + \sin(\alpha_0 - \gamma)\sin(\alpha' - \gamma)$$

$$\cos^2(\alpha' - \gamma) = \tfrac{1}{2}\left(1 + \cos 2(\alpha' - \gamma)\right)$$

$$\sin^2(\alpha' - \gamma) = \tfrac{1}{2}\left(1 - \cos 2(\alpha' - \gamma)\right)$$

and

$$\cos(\alpha' - \gamma)\sin(\alpha' - \gamma) = \tfrac{1}{2}\sin 2(\alpha' - \gamma)$$

which yields the equation:

$$P\cos 2(\alpha' - \gamma) + Q\sin 2(\alpha' - \gamma) = S \qquad (6)$$

where $P = (1 - K)\cos 2(\alpha_0 - \gamma) + K\dfrac{a^2}{R_0^2}$

$Q = (1 - K)\sin 2(\alpha_0 - \gamma)$

$S = 1 - K - (2 - K)\dfrac{a^2}{R_0^2}$.

However, this is recognised to be of a standard trigonometric form enabling us to write:

$$\left(P^2 + Q^2\right)^{\frac{1}{2}} \sin\left\{2(\alpha' - \gamma) + \xi\right\} = S \tag{7}$$

where $\xi = \tan^{-1}\left\{\frac{P}{Q}\right\}$.

Rearranging, we find that the angle of depression of the tangent is given by:

$$\alpha' = \frac{1}{2}\left[\sin^{-1}\left(\frac{S}{\left(P^2 + Q^2\right)^{\frac{1}{2}}}\right) - \xi\right] + \gamma \ . \tag{8}$$

The two tangent angles required are given by the two values of the $\sin^{-1}$ term between 0 and 2π which are, of course, simply related since their sum is equal to π . It follows therefore, that if the tangent angles to the top and bottom of the store are α_1 and α_1 respectively, then:

$$\alpha_1 + \alpha_2 = \frac{\pi}{2} - \xi + 2\gamma \tag{9}$$

and

$$\alpha_1 - \alpha_2 = \frac{\pi}{2} - \sin^{-1}\left(\frac{S}{\left(P^2 + Q^2\right)^{\frac{1}{2}}}\right)$$

$$= \cos^{-1}\left(\frac{S}{\left(P^2 + Q^2\right)^{\frac{1}{2}}}\right) \ . \tag{10}$$

These sum and difference equations may be further developed, as shown in Appendix C, to give the following equations for the range and depression angle of the store centre:

$$\alpha_0 = \frac{1}{2}\left[(\alpha_1 + \alpha_2) - \sin^{-1}\left\{\frac{K}{1 - K}\frac{a^2}{R_0^2} \sin\left\{2\gamma - (\alpha_1 + \alpha_2)\right\}\right\}\right] \tag{11}$$

$$\frac{a^2}{R_0^2} = \frac{\begin{aligned}&2 + K\left\{\cos^2(\alpha_2 - \alpha_1)\cos 2(\alpha_0 - \gamma) - 1\right\} - 2\cos(\alpha_2 - \alpha_1) \times \\ &\times \left\{1 - K\sin^2(\alpha_0 - \gamma)\left\{2 - K + K\cos^2(\alpha_0 - \gamma)\cos^2(\alpha_2 - \alpha_1)\right\}\right\}^{\frac{1}{2}}\end{aligned}}{4 + \frac{K^2}{1 - K}\sin^2(\alpha_2 - \alpha_1)}$$

...... (12)

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The ranges and depression angles of the store centres in the two camera planes will of course be different in general but can be related for a given store attitude by considering the known camera geometry.

2.4 Camera geometry

The two sets of parameters which apply to the two different camera planes may be conveniently distinguished by the use of primed and unprimed symbols. Thus the set of unprimed symbols used up to now $\left\{R_0, \alpha_0, \alpha_1, \alpha_2, \psi_0\right\}$ may be taken as applying to the forward camera, while the corresponding primed symbols $\left\{R_0', \alpha_0', \alpha_1', \alpha_2', \psi_0'\right\}$ refer to the aft camera.

Appendix D shows how the range and depression angles in any two camera planes are related for a given store attitude (θ, ψ) and camera separation D.

Thus:

$$\tan \alpha_0' = \frac{R_0\left\{\sin\theta \cos\alpha_0 \sin(\psi_0' - \psi_0) + \cos\theta \sin\alpha_0 \sin(\psi_0' - \psi)\right\} + D \sin\theta \sin\psi_0'}{\cos\theta\left\{R_0 \cos\alpha_0 \sin(\psi_0 - \psi) - D \sin\psi\right\}} \qquad \text{...... (13)}$$

and

$$R_0' = \frac{R_0 \cos\alpha_0 \sin(\psi_0 - \psi) - D \sin\psi}{\cos\alpha_0' \sin(\psi_0' - \psi)} \ . \qquad (14)$$

The term in α_0' may be eliminated from equation (14) by substitution from equation (13) but this is tedious and gives a very cumbersome result, whereas the above form is perfectly adequate for numerical evaluation, provided that equation (13) is evaluated first.

Appendix D also shows that attitude may be determined from a knowledge of the range and depression angle of the store centre in the two camera planes:

$$\tan\psi = \frac{R_0 \cos\alpha_0 \sin\psi_0 - R_0' \cos\alpha_0' \sin\psi_0'}{D + R_0 \cos\alpha_0 \cos\psi_0 - R_0' \cos\alpha_0' \cos\psi_0'} \qquad (15)$$

$$\tan\theta = \sin\psi\left\{\frac{R_0' \sin\alpha_0' - R_0 \sin\alpha_0}{R_0 \cos\alpha_0 \sin\psi_0 - R_0' \cos\alpha_0' \sin\psi_0'}\right\} . \qquad (16)$$

The term in ψ in equation (16) may be eliminated by substitution from equation (15) but again the above simpler form is adequate for numerical evaluation, provided that ψ is determined first. The second pair of equations is of course equivalent to the first (equations (13) and (14)), having been derived from them merely by a rearrangement of terms, and completes the set of equations necessary for the determination of store position and attitude in elevation and heading. The solution of the equations is discussed in section 3.

It should be noted that equations (13) to (16) inclusive can be divided top and bottom by store radius leaving only normalized dimensions R_0/a and D/a and showing that behaviour can be simply scaled according to store radius.

2.5 Bank angle

Although not required for the present application the measurement of bank angle is presented here for completeness. Its determination requires an *a priori* knowledge of the store position and attitude in heading and elevation, whereupon it may be directly calculated from the observed sight-line angles to marks painted on the side of the store.

It is shown in Appendix E that the sight-line depression angle α_3, in the plane of the forward camera, of longitudinal marks on the side of the store at a bank angle ϕ is given by:

$$\tan \alpha_3 = \frac{\frac{R_0}{a} \sin \alpha_0 \cos \theta \sin(\psi_0 - \psi) - \sin \phi \sin(\psi_0 - \psi) + \sin \theta \cos \phi \cos(\psi_0 - \psi)}{\cos \theta \left\{\frac{R_0}{a} \cos \alpha_0 \sin(\psi_0 - \psi) - \cos \phi\right\}} . \qquad \ldots\ldots (17)$$

This may be rearranged to give bank angle ϕ as a function of the measured angle α_3 :

$$\phi = \eta - \sin^{-1}\left\{\frac{\frac{R_0}{a} \cos \theta \sin(\psi_0 - \psi)\left\{\cos \alpha_0 \tan \alpha_3 - \sin \alpha_0\right\}}{\left[1 + \cos^2\theta\left\{\tan^2\alpha_3 + 2 \tan \alpha_3 \tan \theta \cos(\psi_0 - \psi) - \cos^2(\psi_0 - \psi)\right\}\right]^{\frac{1}{2}}}\right\} \qquad \ldots\ldots (18)$$

where $\eta = \tan^{-1}\left\{\frac{\cos \theta \tan \alpha_3 + \sin \theta \cos(\psi_0 - \psi)}{\sin(\psi_0 - \psi)}\right\}$.

Similar expressions apply to the aft camera of course but with R_0, α_0 and ψ_0 replaced by R_0', α_0' and ψ_0' respectively. The direct calculation of a mean roll angle in this way, is quite straightforward once store position and attitude in elevation and heading are known.

3 SOLVING FOR POSITION AND ATTITUDE

Evaluating equations (11) and (12) in the two camera planes (using primed and unprimed symbols as appropriate), together with a pair of the camera geometry equations from section 2.4, provides a set of six equations altogether, giving the parameters R_0, α_0, R_0', α_0', θ and ψ in terms of each other and known or measured quantities, and which in principle can be solved to find these parameters. An analytical solution of these equations by the conventional technique of substitution and elimination would, however, be extremely tedious even if it were possible and this route has not been seriously considered. Instead a relatively simple iterative technique has been employed which is easily implemented on a programmable calculator or microcomputer and since the computational load is sufficient to require the use of such a machine in any case, this does not present any additional problems. The iterative procedure adopted is described below.

The first step in the procedure is to make an initial estimate of the ranges and depression angles of the store centre in the two camera planes. This is conveniently achieved by assuming that the store section in these planes is circular, *ie* that there is no pitch ($\theta = 0$) and the camera planes are orthogonal to the store axis $\left((\psi_0 - \psi) = (\psi'_0 - \psi) = \pi/2\right)$. In this special case equations (11) and (12) simplify considerably giving, for the forward camera:

$$\alpha_0 = \tfrac{1}{2}(\alpha_1 + \alpha_2) \tag{19}$$

$$\frac{R_0}{a} = \operatorname{cosec}\left(\frac{\alpha_2 - \alpha_1}{2}\right) \tag{20}$$

and similarly for the aft camera:

$$\alpha'_0 = \tfrac{1}{2}(\alpha'_1 + \alpha'_2) \tag{21}$$

$$\frac{R'_0}{a} = \operatorname{cosec}\left(\frac{\alpha'_2 - \alpha'_1}{2}\right) . \tag{22}$$

Using these initial estimates for the positions of the store centres in the two camera planes it is now possible to obtain a more realistic estimate of the store attitude knowing the camera geometry and using equations (15) and (16) which are repeated here in their non-dimensional form.

$$\tan\psi = \frac{\frac{R_0}{a}\cos\alpha_0\sin\psi_0 - \frac{R'_0}{a}\cos\alpha'_0\sin\psi'_0}{\frac{D}{a} + \frac{R_0}{a}\cos\alpha_0\cos\psi_0 - \frac{R'_0}{a}\cos\alpha'_0\cos\psi'_0} \tag{23}$$

$$\tan\theta = \sin\psi\left\{\frac{\frac{R'_0}{a}\sin\alpha'_0 - \frac{R_0}{a}\sin\alpha_0}{\frac{R_0}{a}\cos\alpha_0\sin\psi_0 - \frac{R'_0}{a}\cos\alpha'_0\sin\psi'_0}\right\} . \tag{24}$$

This new estimate of attitude may in turn be used to obtain a better estimate of the true position of the store centres in each of the camera planes. Thus from equations (11) and (12) for the forward camera:

$$\alpha_0 = \frac{1}{2}\left[(\alpha_1 + \alpha_2) - \sin^{-1}\left\{\frac{K}{1-K}\frac{a^2}{R_0^2}\sin\left\{2\gamma - (\alpha_1 + \alpha_2)\right\}\right\}\right] \tag{25}$$

$$\frac{a^2}{R_0^2} = \frac{\begin{aligned}&2 + K\left\{\cos^2(\alpha_2 - \alpha_1)\cos 2(\alpha_0 - \gamma) - 1\right\} - 2\cos(\alpha_2 - \alpha_1)\,\times\\ &\times\left\{1 - K\sin^2(\alpha_0 - \gamma)\left\{2 - K + K\cos^2(\alpha_0 - \gamma)\cos^2(\alpha_2 - \alpha_1)\right\}\right\}^{\frac{1}{2}}\end{aligned}}{4 + \frac{K^2}{1-K}\sin^2(\alpha_2 - \alpha_1)}$$

...... (26)

where $K = 1 - \cos^2\theta \sin^2(\psi_0 - \psi)$

$\gamma = \tan^{-1}\left\{\cot\theta \cos(\psi_0 - \psi)\right\}$

and similarly for the aft camera

$$\alpha_0' = \frac{1}{2}\left[(\alpha_1' + \alpha_2') - \sin^{-1}\left\{\frac{K'}{1 - K'}\frac{a^2}{R_0'^2}\sin\left\{2\gamma' - (\alpha_1' + \alpha_2')\right\}\right\}\right] \tag{27}$$

$$\frac{a^2}{R_0'^2} = \frac{2 + K'\left\{\cos^2(\alpha_2' - \alpha_1')\cos 2(\alpha_0' - \gamma') - 1\right\} - 2\cos(\alpha_2' - \alpha_1') \times \times\left\{1 - K\sin^2(\alpha_0' - \gamma')\left\{2 - K' + K'\cos^2(\alpha_0' - \gamma')\cos^2(\alpha_2' - \alpha_1')\right\}\right\}^{\frac{1}{2}}}{4 + \frac{K'^2}{1 - K'}\sin^2(\alpha_2' - \alpha_1')}$$

...... (28)

where $K' = 1 - \cos^2\theta \sin^2(\psi_0' - \psi)$

$\gamma' = \tan^{-1}\left\{\cot\theta \cos(\psi_0' - \psi)\right\}$.

This procedure may be repeated by the cyclic evaluation of equations (23) to (28) until the desired accuracy is achieved.

A programme to implement the procedure described above has been written for a TI59 programmable calculator and has an execution time of approximately 1 min/iteration. It is estimated that this same programme would run some 7 times faster on a small desk top machine such as the Hewlett Packard HP85.

The number of iterations required before the answer is obtained to a given degree of accuracy depends on the store attitude and position and generally increases markedly with the eccentricity of the store section in the camera planes. It also depends on the orientation of this elliptical section and can require more than twice the number of iterations when the major axis of the ellipse is nominally at right angles to the mean camera sight-line compared to when the major axis is aligned with this direction.

The convergence of the iterative solution is illustrated in Tables 1 and 2 for a particular case being considered where the cameras are separated by only 1.25 store radii (D/a = 1.25) and view at 90° to the datum heading direction. The use of smaller values of D/a than this is unlikely in practice and to this extent this example represents a worst case. The store is assumed to be nominally at a range of five store radii and at a depression angle of 30° but with elevation and heading angles of 10° in the first case and 30° in the second. It is seen that the initial estimates of attitude (iteration number 0 in the tables) are within 0.5° when the elevation and heading angles are 10° but

are in error by up to 7° for the 30° attitude. Convergence is also more rapid in the 10° case with the answer being correct to 0.001° in only two iterations while ten iterations are required in the 30° case. *The data given in the tables for the infinite iteration* case, are in fact the initial data used to calculate the sight-line tangent angles fed into the iterative programme and which were computed using equations presented in this Report.

The convergence behaviour for elevation and heading angles both of opposite sign to those just considered would be identical to that shown above, since the eccentricity and orientation of the store sections would be the same, but convergence would be faster if only one were of different sign, since the ellipse would now be oriented more towards the cameras as discussed earlier. Thus for elevation $\theta = -30^\circ$ and heading $\psi = 30^\circ$ the initial estimates of attitude are within 1.7° and the solution is correct to 0.001° in only four iterations.

4 ERROR ANALYSIS

4.1 General

The foregoing equations may be used to compute the detailed effects of measurement errors occurring in the sight-line tangent angles $(\alpha_1,\alpha_2,\alpha_1',\alpha_2')$ or of errors in setting up the camera geometry (ψ_0,ψ_0',D) for any particular store position and attitude. However it is more useful to the system designer to have simple analytical expressions which relate the resultant errors in position and attitude to the measurement or setting up errors where these can be derived.

An estimate of the effect of finite or limited camera resolution is of particular interest here. In practice resolution may be determined by the size of the individual CCD photosites and the focal length of the lens employed, giving rise to a quantization in the angular measurements and it is important to know how the errors associated with this translate into uncertainties in store attitude and position. The effect of measurement errors has therefore been considered in some detail. Errors in the determination of bank angle have not been considered.

4.2 Measurement errors

Simple analytical expressions for the effect of measurement errors may be found by restricting attention to the moderate pitch and yaw situation, *eg* such that $\sin\theta \approx \tan\theta \approx \theta$, to when the store is some distance away from the cameras $\left((R_0/a)^2 \gg 1\right)$, and to when the cameras are orthogonal to the reference heading direction $(\psi_0 - \psi_0' = \pi/2)$. To a first approximation the effects of measurement errors can then be assumed to be independent of actual store attitude and, more specifically, these effects can be quantified by considering the zero pitch and yaw case, which greatly simplifies the mathematics involved.

The uncertainties in range δR_0 and depression angle $\delta\alpha_0$ can be related to the errors $\delta\alpha_1$ and $\delta\alpha_2$ in the measurement of the sight-line tangent angles α_1 and α_2 *respectively, for the forward camera*, by the partial differential equations for separable variables:

$$\delta R_0 = \frac{\partial R_0}{\partial \alpha_1}\delta\alpha_1 + \frac{\partial R_0}{\partial \alpha_2}\delta\alpha_2 \qquad (29)$$

$$\delta \alpha_0 = \frac{\partial \alpha_0}{\partial \alpha_1}\delta\alpha_1 + \frac{\partial \alpha_0}{\partial \alpha_2}\delta\alpha_2 \quad . \qquad (30)$$

Similar expressions apply to the aft camera using the appropriate primed variables.

The partial derivatives in the above equations may be found in the zero pitch and yaw case ($\theta = 0$, $\psi = 0$) by differentiation of equations (19) and (20), substitution of these yields:

$$\delta R_0 = \frac{R_0^2}{2a}(\delta\alpha_2 - \delta\alpha_1) \qquad (31)$$

and

$$\delta\alpha_0 = \tfrac{1}{2}(\delta\alpha_1 + \delta\alpha_2) \quad . \qquad (32)$$

These equations together with their counterparts for the aft camera enable the mean positional errors in range and depression angle, ΔR_0 and $\Delta\alpha_0$ respectively, to be calculated, using:

$$\Delta R_0 = \tfrac{1}{2}(\delta R_0 + \delta R_0') \qquad (33)$$

$$\Delta\alpha_0 = \tfrac{1}{2}(\delta\alpha_0 + \delta\alpha_0') \quad . \qquad (34)$$

The errors $\delta\psi$ and $\delta\theta$ in the heading and elevation angles respectively, may be determined similarly, from their partial differential equations:

$$\delta\psi = \frac{\partial\psi}{\partial R_0}\delta R_0 + \frac{\partial\psi}{\partial R_0'}\delta R_0' + \frac{\partial\psi}{\partial\alpha_0}\delta\alpha_0 + \frac{\partial\psi}{\partial\alpha_0}\delta\alpha_0' \qquad (35)$$

and

$$\delta\theta = \frac{\partial\theta}{\partial R_0}\delta R_0 + \frac{\partial\theta}{\partial R_0'}\delta R_0' + \frac{\partial\theta}{\partial\alpha_0}\delta\alpha_0 + \frac{\partial\theta}{\partial\alpha_0'}\delta\alpha_0' \quad . \qquad (36)$$

The partial derivatives required here may be obtained from the attitude equations (15) and (16), which simplify in the case being considered, to:

$$\psi \approx \tan\psi \approx \frac{R_0\cos\alpha_0 - R_0'\cos\alpha_0'}{D} \qquad (37)$$

$$\theta \approx \tan\theta \approx \frac{R_0'\sin\alpha_0' - R_0\sin\alpha_0}{D} \quad . \qquad (38)$$

Thus for $\alpha_0' \approx \alpha_0$ and $R_0' \approx R_0$ we have:

$$\delta\psi = \frac{\cos\alpha_0}{D}(\delta R_0 - \delta R_0') + \frac{R_0}{D}\sin\alpha_0(\delta\alpha' - \delta\alpha_0) \qquad (39)$$

$$\delta\theta = \frac{\sin\alpha_0}{D}(\delta R_0' - \delta R_0) + \frac{R_0}{D}\cos\alpha_0(\delta\alpha_0' - \delta\alpha_0) \quad . \tag{40}$$

The positional errors given by equations (31) and (32) and their counterparts for the aft camera may be substituted in the above to give the attitude errors directly in terms of the measurement errors $\delta\alpha_1$, $\delta\alpha_2$, $\delta\alpha_1'$ and $\delta\alpha_2'$.

Of particular interest to the modelling of the effects of finite camera resolution, is the case where all four measurements can be in error by the same amount, of magnitude $\Delta\alpha$ say, but where the sign of the error can be positive or negative in an arbitrary manner. Since there are four measurements there will be 16 different possible error combinations altogether, but only eight of these will lead to different magnitudes of error in store position and attitude, the other eight giving similar magnitudes but opposite signs. It is a relatively simple, if somewhat tedious matter to evaluate these eight different error combinations but only those giving rise to the worst position and attitude errors will be discussed further here.

The largest range error occurs when both measurements to the top of the store (α_1 and α_1') are too large and those to the bottom (α_2 and α_2') are too small, or *vice versa*, giving;

$$\Delta R_0 = \pm\frac{R_0^2\Delta\alpha}{a} \quad . \tag{41}$$

The worst case depression angle error obviously arises when all the measurement errors have the same sign and:

$$\Delta\alpha_0 = \pm\,\Delta\alpha \quad . \tag{42}$$

The largest attitude errors generally occur when the measurement errors combine to make the nose of the store appear closer to the camera than it really is and the tail appear further away, or *vice versa*. Thus, for the forward camera, the top sight-line angle might be too small and that to the bottom too large, while for the aft camera the top angle would be too large and the bottom one too small. The errors in this situation are given by:

$$\delta\psi = \frac{2R_0^2\cos\alpha_0\,\Delta\alpha}{Da} \tag{43}$$

and

$$\delta\theta = \frac{2R_0^2\sin\alpha_0\,\Delta\alpha}{Da} \quad . \tag{44}$$

For purely horizontal viewing ($\alpha_0 = 0$) the error in yaw reaches a maximum but the pitch error $\delta\theta$ is then zero for this particular error format. The worst case pitch error in this situation occurs when both measurement errors for the forward camera are of the same polarity, *eg* making the nose appear high, while those at the tail are both opposite to this, making the tail appear low. The pitch error is then given by:

$$\delta\theta = \frac{2R_0\cos\alpha_0\,\Delta\alpha}{D} \quad . \tag{45}$$

It is easily seen that this exceeds the error given by equation (44) above for small angles, such that $\alpha_0 < \tan^{-1}(a/R_0)$, and this very roughly equates to the store centre being less than one store radius below the horizontal plane containing the camera, *ie* to small drop heights.

Consideration of all 16 possible measurement error combinations gives average position and attitude errors which are less than half of the worst case values presented above. In practice, with a random error distribution, there will be occasions when some or all of the measurement errors are zero and the resultant attitude and position errors will therefore be even smaller. It is common practice in such a situation to equate the worst case errors with three times the standard deviation of the random errors.

The worst case attitude errors can be many times greater than the measurement errors in the sight-line tangent angles as numerical evaluation of equations (43) and (44) quickly shows. Thus in the specific case considered in section 3 ($R_0/a = 5$, $D/a = 1.25$, $\alpha_0 = 30^\circ$) we have:

$$\left.\begin{aligned} \delta\psi &= 34.6\ \Delta\alpha \\ \delta\theta &= 20.0\ \Delta\alpha\ . \end{aligned}\right\} \qquad (46)$$

Hence to measure attitude to $\pm 1^\circ$ in this case it would be necessary to measure the sight-line tangent angles with an error of less than $\pm 0.03^\circ$ and it is clear that very stringent demands can be placed on the system designer in this way.

The major questions posed by the system designer are likely to involve the optimisation of the viewing geometry, *eg* camera spacing and range, in order to minimise the influence of the measurement errors discussed above. The dependencies contained in the error equations indicate that the technique is best suited to long stores, enabling the cameras to be placed a large distance apart (D large) and, at first sight, suggest greatly improved performance at relatively short ranges (R_0 small).

Clearly camera separation must be made as large as practically feasible although it is really the separation of the viewing planes at the store position which is of importance here and so toe-in or out of the cameras can be used as appropriate, with benefit, when the choice of camera position is otherwise constrained.

The apparent dependence of attitude errors on the square of range in equations (43) and (44) can be substantially modified by the interdependencies of depression angle α_0 , angular resolution $\Delta\alpha$ and range R_0 which arise in practice. The designer is usually interested in observing the motion of the store as it falls through a predetermined vertical drop height H and obviously as this is viewed from greater and greater range the sight-line depression angles involved become smaller and hence α_0 which enters into the error equations is also smaller. Furthermore the focal length of the camera lens can be increased as it moves away to maintain the field of view in the plane of the store equal to the required drop height. This also maintains a constant spatial resolution in the store plane, at least to a first approximation, if it is assumed that

resolution is limited by the CCD element size, but angular resolution, $\Delta\alpha$, will then actually improve in inverse proportion to the increase in range R_0.

The result of such interdependencies is that the error in heading angle, $\delta\psi$, now varies in direct proportion to the horizontal separation of the camera and store ($R_0 \cos \alpha_0$) rather than with R_0^2, while the errors in elevation angle $\delta\theta$, given by equation (44) for large α_0, become directly proportional to the vertical separation of the camera and store ($R_0 \sin \alpha_0$). The latter can be minimised by arranging for near horizontal viewing whenever possible and in this event the elevation errors depend only on the vertical drop height H which must be covered. As indicated earlier this will usually be a fixed parameter in the experiments and the worst case elevation angle error then becomes essentially independent of range.

In the small angle case when equation (45) applies ($H < a$) elevation errors will increase with range due to the associated decrease in α_0. This variation will typically be small since α_0 is small anyway.

Different dependencies in range will of course be obtained if the designer invokes different practical links between resolution, range and depression angle to those assumed above. It is left to the reader to evaluate these as necessary. The general conclusions to be reached here however, are that the influence of measurement errors on attitude can be minimised by wide camera separation, near horizontal viewing and, in the typical case considered ($\Delta\alpha \propto 1/R_0$), by minimising viewing range, although the benefits attributable to the latter constraint are less than would appear at first sight to be the case.

Changing the camera separation D or nominal viewing angle α_0 is seen to have no effect on the magnitude of the worst cases position errors (ΔR_0 and $\Delta\alpha_0$) arising from measurement errors in the sight-line tangent angles, as might be expected. In discussing the influence of viewing range on these position errors however, we may again invoke practical relationships between range depression angle and angular resolution, thus modifying the apparent dependencies given in equations (41) and (42). Thus if we again vary lens focal length to give an angular resolution $\Delta\alpha$ inversely proportional to range, we find that the error in range ΔR_0 is now directly proportional to the range R_0, while the angular error $\Delta\alpha_0$ varies inversely with this. The net effect is that positional uncertainty along the line of sight is proportional to range while uncertainty at right angles to the sight-line, given by the product $R_0\Delta\alpha_0$, is essentially constant. For near horizontal viewing conditions these directions may be roughly equated with the local horizontal and vertical respectively so that in this case horizontal position error will increase in direct proportion to viewing range, while the uncertainty in height remains constant. Once again therefore the overall balance is probably in favour of shorter viewing ranges.

The influence of store radius on the effects of measurement errors is seen in equations (41), (43) and (44) which indicate a reduction in range and attitude errors for large radius stores, all other things being equal. Physically this goes together with the requirement for shorter viewing ranges noted above and the ability, observed

in section 2, to normalise all the dimensions involved in the geometrical equations by store radius, which in themselves would suggest that normalised range R_0/a is the important parameter here and the one to be minimised.

4.3 Setting up errors

The effects of errors in setting up the camera geometry, both in attitude and separation, may be discussed qualitatively in general, or computed accurately for specific cases using the iterative technique described earlier.

Errors in the camera separation D would not be expected to lead to significant errors in store position, at least to a first order approximation when near orthogonal viewing conditions are used. However errors in store attitude can be generated and these would be expected to vary in direct proportion to the errors in D . Since the latter are easily limited in typical practical situations to much less than 1%, this error source can be effectively eliminated.

The effects of errors in the camera heading angles (ψ_0 and ψ_0') can, however, be much more serious, especially when the camera separation is small and the viewing range large, since, as indicated earlier, it is really the separation between the camera planes at the store which is important in determining attitude. It is easy to deduce an expression for the fractional change in store attitude resulting from a change $\delta\psi_0$ in both camera heading angles, the worst case being when both cameras angle in towards each other or both toe out. To a first approximation the attitude errors would then be expected to be given by:

$$\frac{\delta\theta}{\theta} = \frac{\delta\psi}{\psi} = \frac{2R_0 \cos\alpha_0}{D}\delta\psi_0 \quad . \tag{47}$$

These errors can obviously be minimised by making the camera separation D as large as possible and selecting a small horizontal working distance ($R_0 \cos\alpha_0$). In the particular case considered earlier, where $R_0/a = 5$, $D/a = 1.25$ and $\alpha_0 = 30^\circ$, equation (47) shows that to measure attitude to $\pm 1^\circ$ over a range of $\pm 20^\circ$ in store elevation and heading, camera heading angle must be known to better than $\pm 0.4^\circ$.

Camera heading angle errors would be expected to have a negligible effect on store position, to a first approximation, for near orthogonal viewing conditions $\left((\psi_0 - \psi) \approx (\psi_0' - \psi) \approx \pi/2\right)$.

Errors in the camera elevation angles will lead to errors in the determination of the store depression angle α_0 if these are in the same direction for both cameras and to errors in store attitude if of opposite polarity. The principal attitude error would be in elevation for near horizontal viewing. There is of course a close parallel here with some of the measurement error combinations discussed in section 4.2 and similar equations are obtained for modelling purposes. Thus, by analogy with equations (42) and (45), the worst case depression angle error $\Delta\alpha_0$ resulting from a camera elevation angle error $\delta\Theta$ is:

$$\Delta\alpha_0 = \pm\,\delta\Theta$$

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which for near horizontal viewing will mainly result in an error in store height measurement, and the worst case store elevation angle error $\delta\theta$ is given by:

$$\delta\theta = \frac{2R_0 \cos \alpha_0}{D} \delta\Theta \ .$$

The associated store heading angle error is:

$$\delta\psi = \frac{2R_0 \sin \alpha_0}{D} \delta\Theta$$

which tends to zero for horizontal viewing ($\alpha_0 \approx 0$).

The resultant errors in attitude and height measurement will all increase with greater viewing range and the attitude errors will also become worse for smaller camera separations. For $R_0/a = 5$, $D/a = 1.25$ and $\alpha_0 = 30^\circ$ the elevation and heading errors become respectively seven and four times the error in camera elevation angle and hence for attitude to be determined to $\pm 1^\circ$ camera elevation angle must be accurate to $\pm 0.14^\circ$.

Finally, uncertainties in the radius of the store would be expected to result mainly in proportional errors in the store range with a minimal effect on attitude, any taper in the store profile between the two camera planes would, however, be much more serious in this respect.

4.4 Discussion

It is clear from the error equations presented in sections 4.2 and 4.3 that the effects of both measurement errors and setting up errors can be minimised in general by working at relatively close range and with as large a separation of the camera planes as possible. While the effects of measurement errors will usually be random in nature, setting up errors will be systematic and are therefore less of a problem in practice since they can be largely eliminated by calibration and they do not in any case significantly affect the measurement of store velocities or angular rates which are frequently the variables of prime interest.

Calculations of the errors in specific cases, using the iterative techniques described in section 3, has confirmed the general validity of the error equations presented above. The agreement obtained is excellent except in the single case of the effect of camera heading angle errors on store attitude (equation (47)). The effects of pitch and yaw interact markedly here but even so the simple formula presented holds with ±50% for elevation, heading and depression angles in the range $\pm 30^\circ$ and this is adequate for most design purposes.

There are of course many other aspects which must be considered in arriving at an overall specification of system errors such as the problems of distortion in the optics, lens focus, depth of field and resolution and vibration. These however, are beyond the scope of this Report.

5 CONCLUSIONS

The derivation of the equations relating the position and attitude of a cylindrical object to the sight-line tangent angles measured in two vertical planes has been presented together with an iterative technique for their solution. The convergence of the iterative technique has been illustrated by results obtained with a programme written for a TI59 calculator.

Simple first order approximations have been derived for the dependence of attitude and position errors on both measurement errors and errors in setting up the camera geometry. It is shown that the resultant attitude errors can be very many times greater than the measurement errors causing them and that the technique is best suited to long stores, enabling a wide separation of the camera planes, and for relatively short range viewing (R_0/a small).

Appendix A

THE EQUATION OF THE STORE SURFACE

Consider the point P on the surface of a cylindrical store of radius a with coordinates $R\alpha\beta$ in the coordinate system shown in Fig 2 and having its origin at a point O on the axis of the store. Angle α represents the elevation angle of point P above the horizontal XY plane while β is the azimuth angle measured in this horizontal plane relative to the X axis.

The elevation and azimuth angles defining the attitude of the store are θ and ψ respectively, in accordance with convention.

Point C on the axis of the store is the centre of the normal circular section whose circumference contains the point P .

Thus

$$PC = a \tag{A-1}$$

$$OP = R \tag{A-2}$$

and by Pythagoras:

$$OC = \left(R^2 - a^2\right)^{\frac{1}{2}} \quad . \tag{A-3}$$

CD and PA are perpendiculars from C and P respectively, to the horizontal XY plane, CB is the perpendicular from C to PA as shown in Fig 2.

Thus

$$AB = CD = \left(R^2 - a^2\right)^{\frac{1}{2}} \sin\theta \tag{A-4}$$

$$PA = R\sin\alpha \tag{A-5}$$

and

$$PB = PA - AB = R\sin\alpha - \left(R^2 - a^2\right)^{\frac{1}{2}} \sin\theta \quad . \tag{A-6}$$

In triangle DOA

$$OD = \left(R^2 - a^2\right)^{\frac{1}{2}} \cos\theta \tag{A-7}$$

$$OA = R\cos\theta \tag{A-8}$$

$$D\hat{O}A = \beta - \psi \quad . \tag{A-9}$$

By the cosine rule

$$AD^2 = OD^2 + OA^2 - 2 \,.\, OD \,.\, OA \,\cos(D\hat{O}A) \,. \tag{A-10}$$

But

$$AD = BC \tag{A-11}$$

and, by Pythagoras:

$$BC^2 + PB^2 = a^2 \tag{A-12}$$

hence

$$\left\{R \sin\alpha - \left(R^2 - a^2\right)^{\frac{1}{2}} \sin\theta\right\}^2 + (R^2 - a^2)\cos^2\theta + R^2\cos^2\theta$$

$$- 2R\left(R^2 - a^2\right)^{\frac{1}{2}} \cos\alpha\cos\theta\cos(\beta - \psi) = a^2 \qquad \text{(A-13)}$$

$$(R^2 - a^2) - 2R\left(R^2 - a^2\right)^{\frac{1}{2}}\left\{\sin\alpha\sin\theta + \cos\alpha\cos\theta\cos(\beta - \psi)\right\} = 0 \qquad \text{(A-14)}$$

and for $R \neq a$:

$$R^2 - a^2 = R^2\left\{\sin\alpha\sin\theta + \cos\alpha\cos\theta\cos(\beta - \psi)\right\}^2 \qquad \text{(A-15)}$$

or, rearranging, we have for the equation of the surface of the store:

$$R^2 = \frac{a^2}{1 - \left\{\sin\alpha\sin\theta + \cos\alpha\cos\theta\cos(\beta - \psi)\right\}^2} \ . \qquad \text{(A-16)}$$

Appendix B

THE ELLIPSE

The familiar equation of an ellipse in Cartesian coordinates is:

$$\frac{x^2}{p^2} + \frac{y^2}{q^2} = 1 \tag{B-1}$$

where p and q are the magnitudes of the semi-axes in the x and y directions respectively.

Transforming to polar coordinates by means of the substitutions

$$x = R \cos \Phi \tag{B-2}$$

$$y = R \sin \Phi \tag{B-3}$$

gives the polar form of the ellipse:

$$\frac{R^2 \cos^2\Phi}{p^2} + \frac{R^2 \sin^2\Phi}{q^2} = 1 \tag{B-4}$$

which can be rearranged to give:

$$R^2 = \frac{p^2}{1 - \left(1 - \frac{p^2}{q^2}\right)\sin^2\Phi} \;. \tag{B-5}$$

If we now rotate this coordinate system through an angle Ω such that the angle α measured in the new system is given by

$$\alpha = \Phi - \Omega \tag{B-6}$$

we have

$$R^2 = \frac{p^2}{1 - \left(1 - \frac{p^2}{q^2}\right)\sin^2(\alpha + \Omega)} \;. \tag{B-7}$$

This is seen to have exactly the same form as equation (2) of the main text which therefore represents an ellipse.

Comparing coefficients we have

$$a = p \tag{B-8}$$

$$K = 1 - \frac{p^2}{q^2} \tag{B-9}$$

$$\Omega = \gamma \;. \tag{B-10}$$

Hence

$$q = \frac{a}{(1 - K)^{2}} \tag{B-11}$$

and since

$$0 < K < 1$$

$$\frac{a}{(1 - K)^{\frac{1}{2}}} > a$$

then equation (2) therefore represents an ellipse with minor semi-axis a and major semi-axis $a/(1 - K)^{\frac{1}{2}}$, rotated through an angle γ .

Appendix C

DEVELOPMENT OF THE SUM AND DIFFERENCE EQUATIONS

C.1 The sum equation

From the sum of the sight-line tangent angles to the top and bottom of the store given in equation (9) of the main text we have:

$$\tan(\alpha_1 + \alpha_2) = \tan\left\{\left(\frac{\pi}{2} - \zeta\right) + 2\gamma\right\} \tag{C-1}$$

where $$\zeta = \tan^{-1}\left\{\frac{(1 - K)\cos 2(\alpha_0 - \gamma) + K\dfrac{a^2}{R_0^2}}{(1 - K)\sin 2(\alpha_0 - \gamma)}\right\}$$

expanding:

$$\tan(\alpha_1 + \alpha_2) = \frac{\cot\zeta + \tan 2\gamma}{1 - \cot\zeta\tan 2\gamma} \tag{C-2}$$

$$= \frac{(1 - K)\left\{\sin 2(\alpha_0 - \gamma)\cos 2\gamma + \cos 2(\alpha_0 - \gamma)\sin 2\gamma\right\} + K\dfrac{a^2}{R_0^2}\sin 2\gamma}{(1 - K)\left\{\cos 2(\alpha_0 - \gamma)\cos 2\gamma - \sin 2(\alpha_0 - \gamma)\sin 2\gamma\right\} + K\dfrac{a^2}{R_0^2}\cos 2\gamma} \tag{C-3}$$

$$= \frac{\sin 2\alpha_0 + \dfrac{K}{1 - K}\dfrac{a^2}{R_0^2}\sin 2\gamma}{\cos 2\alpha_0 + \dfrac{K}{1 - K}\dfrac{a^2}{R_0^2}\sin 2\gamma} \; . \tag{C-4}$$

Hence rearranging:

$$\frac{\sin(\alpha_1 + \alpha_2)\cos 2\alpha_0 - \cos(\alpha_1 + \alpha_2)\sin 2\alpha_0}{\cos(\alpha_1 + \alpha_2)} = \frac{K}{1 - K}\frac{a^2}{R_0^2}\cos 2\gamma\left\{\tan 2\gamma - \tan(\alpha_1 + \alpha_2)\right\}$$

$$\ldots\ldots \text{(C-5)}$$

$$\sin\left\{(\alpha_1 + \alpha_2) - 2\alpha_0\right\} = \frac{K}{1 - K}\frac{a^2}{R_0^2}\sin\left\{2\gamma - (\alpha_1 + \alpha_2)\right\} \tag{C-6}$$

and

$$\alpha_0 = \frac{1}{2}\left[(\alpha_1 + \alpha_2) - \sin^{-1}\left\{\frac{K}{1 - K}\frac{a^2}{R_0^2}\sin\left\{2\gamma - (\alpha_1 + \alpha_2)\right\}\right\}\right] \; . \tag{C-7}$$

C.2 The difference equation

From the difference between the sight-line tangent angles given in equation (10) of the main text we have:

$$\cos^2(\alpha_2 - \alpha_1) = \frac{S^2}{P^2 + Q^2} \tag{C-8}$$

$$= \frac{(1 - K)^2 + (2 - K)^2 \dfrac{a^4}{R_0^4} - 2(2 - K)(1 - K) \dfrac{a^2}{R_0^2}}{(1 - K)^2 + 2K(1 - K) \cos 2(\alpha_0 - \alpha) \dfrac{a^2}{R_0^2} + K^2 \dfrac{a^4}{R_0^4}} \tag{C-9}$$

which yields a quadratic in a^2/R_0^2:

$$\frac{a^4}{R_0^4}\left\{4(1 - K) + K^2 \sin^2(\alpha_2 - \alpha_1)\right\} - 2 \frac{a^2}{R_0^2} (1 - K)\left\{2 + K\left(\cos^2(\alpha_2 - \alpha_1) \cos 2(\alpha_0 - \gamma) - 1\right)\right\}$$
$$+ (1 - K)^2 \sin^2(\alpha_2 - \alpha_1) = 0 \ . \tag{C-10}$$

Solving the quadratic, choosing the correct root by inspection, and after some tedious manipulation we obtain:

$$\frac{a^2}{R_0^2} = \frac{\begin{array}{l} 2 + K\left\{\cos^2(\alpha_2 - \alpha_1) \cos 2(\alpha_0 + \gamma) - 1\right\} - 2 \cos(\alpha_2 - \alpha_1) \times \\ \times \left\{1 - K \sin^2(\alpha_0 - \gamma)\left\{2 - K + K \cos^2(\alpha_0 - \gamma) \cos^2(\alpha_0 - \alpha_1)\right\}\right\}^{\frac{1}{2}} \end{array}}{4 + \dfrac{K^2}{1 - K} \sin^2(\alpha_2 - \alpha_1)}$$

...... (C-11)

Appendix D

CAMERA GEOMETRY

Fig 4 shows the basic geometry and relative positions of the store centres in the two camera planes. The camera separation is D and the separation of the store centres measured along the axis of the store is L .

Resolving into the horizontal and vertical planes as shown in Fig 4 we have, from the horizontal plane:

$$L \cos \theta \cos \psi = D + R_0 \cos \alpha_0 \cos \psi_0 - R_0' \cos \alpha_0' \cos \psi_0' \tag{D-1}$$

and

$$L \cos \theta \sin \psi = R_0 \cos \alpha_0 \sin \psi_0 - R_0' \cos \alpha_0' \sin \psi_0' \tag{D-2}$$

and from the vertical plane:

$$L \sin \theta = R_0' \sin \alpha_0' - R_0 \sin \alpha_0 \quad . \tag{D-3}$$

Eliminating L from (D-1) and (D-2) gives:

$$\tan \psi = \frac{R_0 \cos \alpha_0 \sin \psi_0 - R_0' \cos \alpha_0' \sin \psi_0'}{D + R_0 \cos \alpha_0 \cos \psi_0 - R_0' \cos \alpha_0' \cos \psi_0'} \quad . \tag{D-4}$$

Similarly from (D-2) and (D-3)

$$\tan \theta = \sin \psi \left\{ \frac{R_0' \sin \alpha_0' - R_0 \sin \alpha_0}{R_0 \cos \alpha_0 \sin \psi_0 - R_0' \cos \alpha_0' \sin \psi_0'} \right\} \tag{D-5}$$

where the $\sin \psi$ term on the right hand side may be eliminated by substitution using equation (D-4) if required.

Equations (D-4) and (D-5) give the store attitude in terms of the camera geometry and positions of the store centres and they may be rearranged to give the relationship between the positions of the store centres for a known attitude.

Rearranging (D-5) we have:

$$R_0' = R_0 \left\{ \frac{\sin \theta \cos \psi_0 \cos \alpha_0 + \cos \theta \sin \psi \sin \alpha_0}{\sin \theta \cos \psi_0' \cos \alpha_0' + \cos \theta \sin \psi \sin \alpha_0'} \right\} \tag{D-6}$$

and from (D-4)

$$D \sin \psi + R \cos \alpha_0 (\cos \psi_0 \sin \psi - \sin \psi_0 \cos \psi)$$

$$= R' \cos \alpha_0' (\cos \psi_0' \sin \psi - \sin \psi_0' \cos \psi) \quad . \tag{D-7}$$

This simplifies to give:

$$R_0' = \frac{R_0 \cos \alpha_0 \sin(\psi_0 - \psi) - D \sin \psi}{\cos \alpha_0' \sin(\psi_0' - \psi)} \quad . \tag{D-8}$$

Equating the right hand sides of equations (D-6) and (D-8) and collecting together terms in α_0' yields:

$$\tan \alpha_0' = \frac{R_0\left\{\sin \theta \cos \alpha_0 \sin(\psi_0' - \psi_0) + \cos \theta \sin \alpha_0 \sin(\psi_0' - \psi)\right\} - D \sin \theta \sin \psi_0'}{\cos \theta\left\{R_0 \cos \alpha_0 \sin(\psi_0 - \psi) - D \sin \psi\right\}} \quad .$$

...... (D-9)

Appendix E

DETERMINATION OF BANK ANGLE

Consider a line painted along the port side of the cylindrical store (the side assumed to be nearest to the cameras), so that in straight and level flight the line lies in the horizontal plane containing the centre line of the store. The problem is to determine the sight-line angle of the painted mark, in either camera plane, when the store banks through an angle ϕ defined in the conventional sense (port wing high), and is otherwise of known attitude and position.

A procedure may be followed which is similar to that adopted in determining the sight-line tangent angles to the top and bottom edges of the store. The general equation of the painted line in three-dimensional space is found first and the intersection of this line with the camera viewing plane then gives the coordinates of the observed point where sight-line angle is required. The foregoing is carried out in a polar coordinate system centred on the store centre line and a simple transformation of axes to a new origin at the camera position enables the sight-line angle to be found in the coordinate system used for experimental measurements.

The geometry of the situation is shown in Fig 5 where P is a point on the line at coordinates $R\alpha\beta$, α and β being the elevation and azimuth of the point P respectively. C is the centre of the normal section having P on its circumference and CH is the horizontal from C in the same normal section of the store.

PA and CD are the perpendiculous from P and C respectively on to the horizontal XY plane containing the origin O , which, as indicated earlier lies on the centre line of the store (OC). Also, PQ is the perpendicular from P to CH , giving $A\hat{P}Q = \theta$, AE is the perpendicular from A to OD , QJ is mutually perpendicular to PQ and CH with J lying on PA , and JK is the perpendicular from J to OC .

Now, since QJ and CK are both mutually perpendicular to the normal section PHC , QJKC is a rectangle and therefore JK is equal to QC and also horizontal. It follows that JKAE is also a rectangle. We have:

$$PA = R \sin \alpha = PJ + AJ \tag{E-1}$$

$$= PQ \sec \theta + KE \tag{E-2}$$

$$= a \sin \phi \sec \theta + OE \tan \theta \tag{E-3}$$

$$= \sec \theta \{a \sin \phi + OA \cos(\psi - \beta) \sin \theta\} \tag{E-4}$$

therefore

$$R \sin \alpha = \sec \theta \{a \sin \phi + R \cos \alpha \cos (\psi - \beta) \sin \theta\} \ . \tag{E-5}$$

Also:

$$AE = OA \sin(\psi - \beta) = JK = QC \tag{E-6}$$

hence

$$R \cos \alpha \sin(\psi - \beta) = a \cos \phi \ . \tag{E-7}$$

Substituting from (E-7) for $R \cos \alpha$ into (E-5) gives two independent equations in R and α which may be solved for these parameters if required, however this is not necessary for present purposes

$$R \sin \alpha = \frac{a\left\{\sin \phi \sin(\psi - \beta) + \sin \theta \cos \phi \cos(\psi - \beta)\right\}}{\cos \theta \sin(\psi - \beta)} \tag{E-8}$$

$$R \cos \alpha = \frac{a \cos \phi}{\sin(\psi - \beta)} . \tag{E-9}$$

As in determining the equation of the store section in the camera plane (section 2.2) we now assume that the camera plane passes through the origin with azimuth ψ_0, enabling the coordinates of the mark observed in the camera plane to be determined by substituting $\beta = \psi_0 + \pi$ in equations (E-8) and (E-9) above for the port side, $\beta = \psi_0$ giving the starboard side. If we also change the coordinate system as before, giving a new origin at the camera position and specifying the new coordinates of point P as range R_3 and depression angle α_3, we have for the port side

$$R_3 \cos \alpha_3 = R_0 \cos \alpha_0 - R \cos \alpha \tag{E-10}$$

$$R_3 \sin \alpha_3 = R_0 \sin \alpha_0 - R \sin \alpha \tag{E-11}$$

where R_0 and α_0 are the range and depression angle of the store centre in the new coordinate system.

Hence:

$$\tan \alpha_3 = \frac{R_0 \sin \alpha_0 - R \sin \alpha}{R_0 \cos \alpha_0 - R \cos \alpha}$$

and substituting from equations (E-8) and (E-9)

$$\tan \alpha_3 = \frac{\frac{R_0}{a} \sin \alpha_0 \cos \theta \sin(\psi_0 - \psi) - \sin \phi \sin(\psi_0 - \psi) + \sin \theta \cos \phi \cos(\psi_0 + \psi)}{\cos \theta \left\{\frac{R_0}{a} \cos \alpha_0 \sin(\psi_0 - \psi) - \cos \phi\right\}} . \tag{E-12}$$

Note that R_0 has been normalized by the store radius a and the angle $(\psi - \psi_0)$ changed to $(\psi_0 - \psi)$ to agree with previous form.

Equation (E-12) may be rearranged to give the bank angle ϕ as a function of the sight-line angle α_3 by cross multiplying and then collecting together terms in $\sin \phi$ and $\cos \phi$, thus:

$$\left\{\cos \theta \tan \alpha_3 + \sin \theta \cos(\psi_0 - \psi)\right\} \cos \phi - \sin(\psi_0 - \psi) \sin \phi$$
$$= \frac{R_0}{a} \cos \theta \sin(\psi_0 - \psi) \left\{\tan \alpha_3 \cos \alpha_0 - \sin \alpha_0\right\} \quad \text{......} \tag{E-13}$$

now

$$U\cos\phi - V\sin\phi = \left(U^2 + V^2\right)^{\frac{1}{2}} \sin(\eta - \phi) \qquad \text{(E-14)}$$

where $\eta = \tan^{-1}\left(\frac{U}{V}\right)$

hence

$$\sin(\eta - \phi) = \left\{\frac{\frac{R_0}{a}\cos\theta\sin(\psi_0 - \psi)\left\{\tan\alpha_3\cos\alpha_0 - \sin\alpha_0\right\}}{\left[1 + \cos^2\theta\left\{\tan^2\alpha_3 + 2\tan\alpha_3\tan\theta\cos(\psi_0 - \psi) - \cos^2(\psi_0 - \psi)\right\}\right]^{\frac{1}{2}}}\right\}$$

...... (E-15)

and, finally:

$$\phi = \eta - \sin^{-1}\left\{\frac{\frac{R_0}{a}\cos\theta\sin(\psi_0 - \psi)\left\{\tan\alpha_3\cos\alpha_0 - \sin\alpha_0\right\}}{\left[1 + \cos^2\theta\left\{\tan^2\alpha_3 + 2\tan\alpha_3\tan\theta\cos(\psi_0 - \psi) - \cos^2(\psi_0 - \psi)\right\}\right]^{\frac{1}{2}}}\right\}$$

...... (E-16)

where $\eta = \tan^{-1}\left\{\frac{\cos\theta\tan\alpha_3 + \sin\theta\cos(\psi_0 - \psi)}{\sin(\psi_0 - \psi)}\right\}$.

Table 1

CONVERGENCE OF ITERATIVE PROGRAMME FOR 10^{o} PITCH AND YAW

Input data

$\frac{D}{a}$	= 1.25	α_1	= 16.55010052
ψ_0	= 90^{o}	α_2	= 40.07081796
ψ_0'	= 90^{o}	α_1'	= 19.82745368
		α_2'	= 43.74644789

Results

Iteration number	Parameter					
	θ	ψ	α_0	$\frac{R_0}{a}$	α_0'	$\frac{R_0'}{a}$
0	9.627474	9.870665	28.310459	4.906320	31.786951	4.825785
1	9.984904	9.984410	28.270516	5.034012	31.753374	4.955189
2	9.998921	9.996105	28.272225	5.041355	31.755280	4.962412
3	9.999955	9.999954	28.272213	5.041757	31.755267	4.962818
4	9.999997	9.999999	28.272218	5.041779	31.755273	4.962840
∞	10.0	10.0	28.272218	5.041780	31.755273	4.962841

Table 2

CONVERGENCE OF ITERATIVE PROGRAMME FOR 30° PITCH AND YAW

Input data

$\frac{D}{a} = 1.25$		$\alpha_1 = 9.814760222$	
$\psi_0 = 90^\circ$		$\alpha_2 = 38.9993655$	
$\psi'_0 = 90^\circ$		$\alpha'_1 = 21.01570820$	
		$\alpha'_2 = 51.88559048$	

Results

Iteration number	Parameter					
	θ	ψ	α_0	$\frac{R_0}{a}$	α'_0	$\frac{R'_0}{a}$
0	23.178612	25.349156	24.407063	3.969209	36.450649	3.757364
1	28.105158	27.478669	23.857871	4.646414	36.174493	4.458827
2	29.074128	29.269900	23.953318	4.937227	36.314031	4.730071
3	29.704085	29.595812	23.935244	5.051800	36.294045	4.847901
4	29.847944	29.877479	23.94[illegible]51	5.099939	36.309429	4.892836
5	29.950265	29.932039	23.944922	5.119082	36.306468	4.912509
6	29.974322	29.979157	23.946859	5.127210	36.308892	4.920093
7	29.991542	29.988483	23.946448	5.130457	36.308409	4.923427
8	29.995643	29.996446	23.946770	5.131837	36.308811	4.924716
9	29.998559	29.998045	23.946702	5.132389	36.308731	4.925282
10	29.999260	29.999394	23.946756	5.132624	36.308799	4.925502
∞	30.0	30.0	23.946753	5.132785	36.308797	4.925627

LIST OF SYMBOLS

a	store radius
D	camera separation
H	store vertical drop height
K	parameter defined by equation (2)
K'	as K but referred to aft camera plane (see equation (28))
p	semi-axis of ellipse
P	variable expression defined by equation (6)
q	semi-axis of ellipse
Q	variable expression defined by equation (6)
R	radius from origin on store centre line
R'	radius from origin at camera position
R_0	range of store centre from camera position
R'_0	as R_0 but referred to aft camera plane
R_3	range of mark on store side from camera position
S	variable expression defined by equation (6)
U	general coefficient of $\cos \phi$ in equation (E-14)
V	general coefficient of $\sin \phi$ in equation (E-14)
α	general elevation angle from origin at store centre
α'	general depression angle from origin at camera
α_0	depression angle of store centre from camera position
α'_0	as α_0 but in aft camera plane
α_1	depression angle of sight-line tangent to top of store from camera position
α'_1	as α_1 but in aft camera plane
α_2	depression angle of sight-line tangent to bottom of store from camera position
α'_2	as α_2 but in aft camera plane
α_3	depression angle of mark on camera side from camera position
β	general azimuth angle from origin at store centre measured relative to direction of line joining the two cameras
γ	parameter defined by equation (2)
γ'	as γ but referred to aft camera plane (see equation (28))
δR_0	increment in range R_0
$\delta R'_0$	increment in range R'_0

LIST OF SYMBOLS (concluded)

$\delta\alpha_0$	increment in depression angle α_0
$\delta\alpha_0'$	increment in depression angle α_0'
$\delta\alpha_1$	increment in depression angle α_1
$\delta\alpha_1'$	increment in depression angle α_1'
$\delta\alpha_2$	increment in depression angle α_2
$\delta\alpha_2'$	increment in depression angle α_2'
$\delta\theta$	increment in store elevation angle θ
$\delta\Theta$	increment in camera elevation angle
$\delta\psi$	increment in store heading angle ψ
$\delta\psi_0$	increment in camera heading angle ψ_0
$\delta\psi_0'$	increment in aft camera heading angle ψ_0'
ΔR_0	mean increment in store range
$\Delta\alpha_0$	mean increment in store depression angle
η	angle defined by equation (E-14)
θ	store elevation angle
ζ	angle defined by equation (7)
ϕ	store bank angle
Φ	polar coordinate defined by equations (B-2) and (B-3)
ψ	store heading angle
ψ_0	forward camera heading angle
ψ_0'	aft camera heading angle
Ω	angle used in equation (B-6)

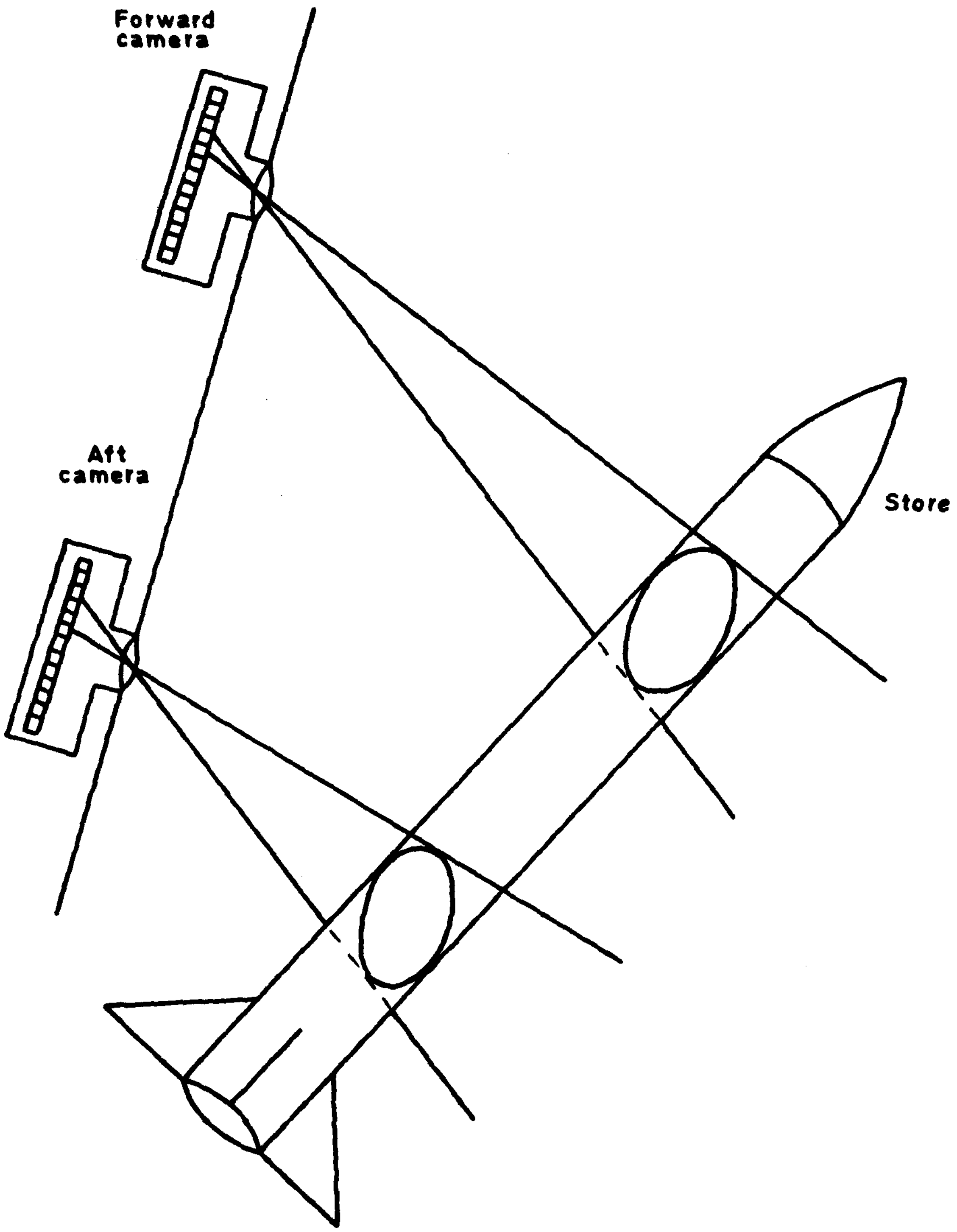

Fig 1 General arrangement

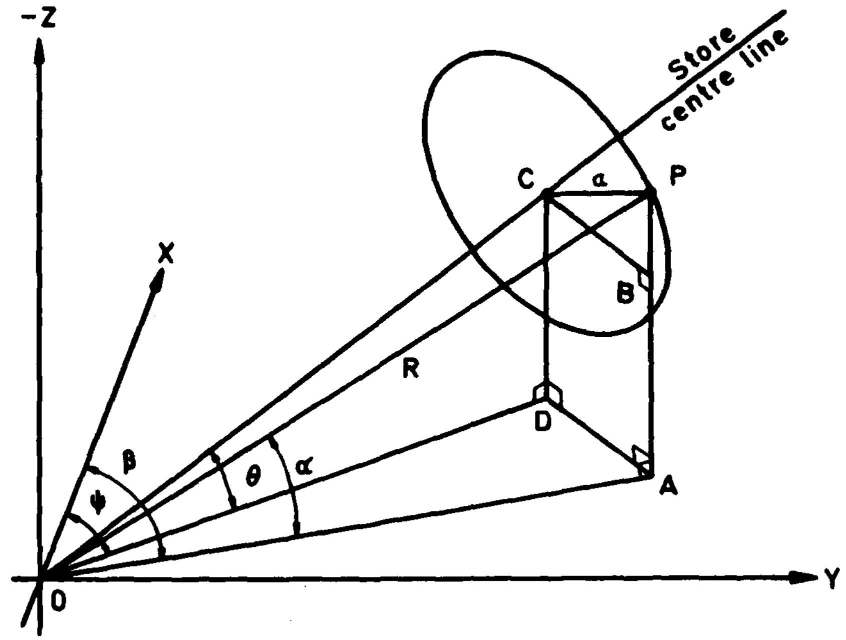

Fig 2 Geometry for determining the equation of the store surface

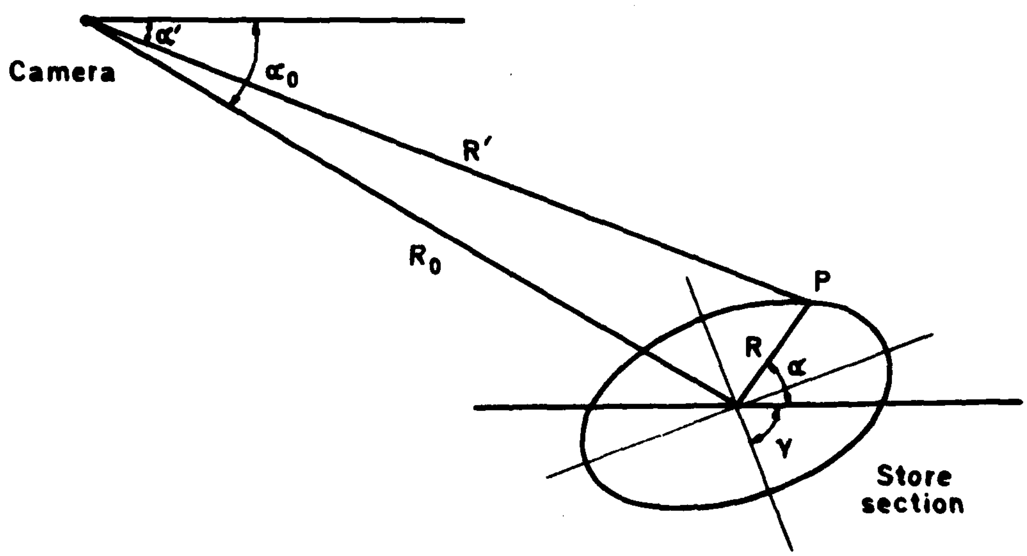

Fig 3 Changing the origin

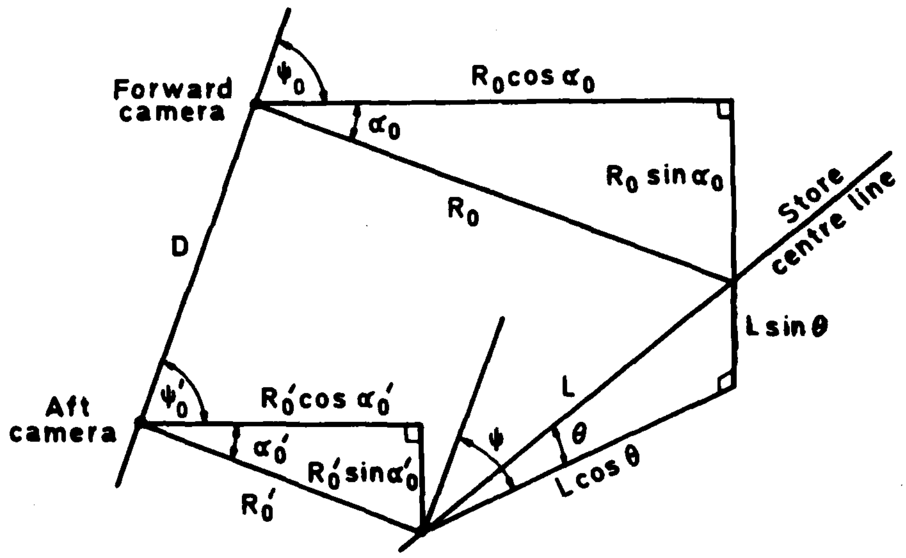

Fig 4 Camera geometry

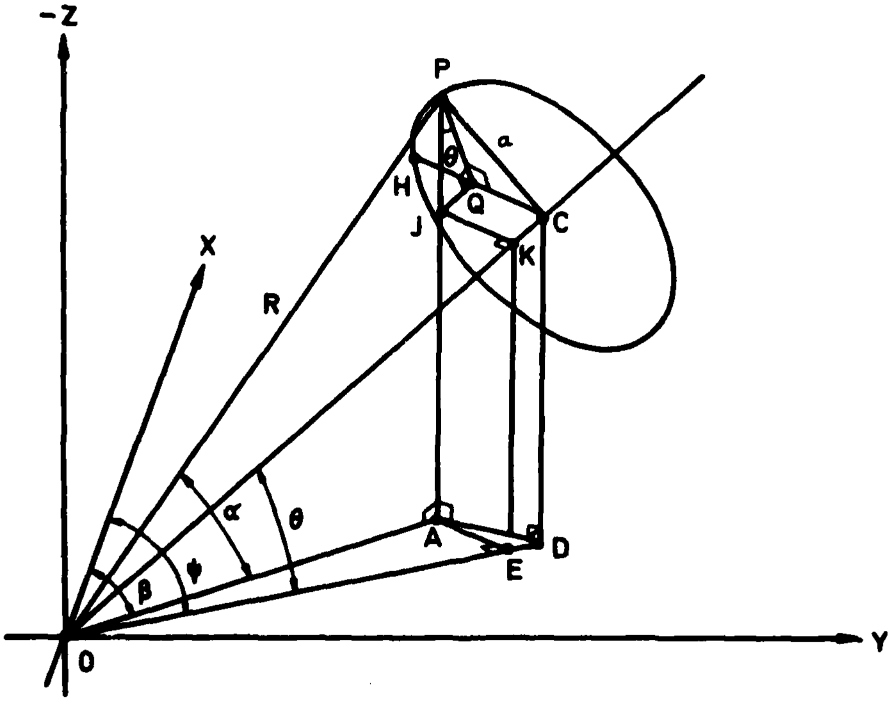

Fig 5 Geometry for determining bank angle

REPORT DOCUMENTATION PAGE

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17. Abstract

The theory is presented of a technique for measuring the attitude and position of cylindrical objects which is of interest in studying the behaviour of stores ejected from aircraft wing pylons. The sight-line angles of the tangents to the top and bottom edges of the store are measured in two vertical planes using linear CCD array cameras, and equations relating these angles to the required attitude and position are derived in this Report. The measurement of bank angle from the observation of marks painted on the side of the store, is also considered. The errors in position and attitude determination, which arise from uncertainties in the angle measurements and camera geometry, are discussed and simple first order approximations are derived for these.